

Space Vector Control of a Three-Phase Rectifier using PLECS®

Dr. John Schönberger
Plexim GmbH
Technoparkstrasse 1
8005 Zürich

1 Introduction

Space vector control is popular for controlling motor drives or three-phase rectifiers since it offers reduced switching losses and better utilization of the DC bus compared to conventional PWM modulation. This report describes a space vector controller for a three-phase boost-type rectifier that is implemented in PLECS. The schematic diagram of the boost rectifier is shown in Fig. 1. The source is a 50 Hz supply and the load is a constant current source that behaves like an infinite inductor.

2 Space Vector Control

The control goal for the three-phase boost rectifier is to generate sinusoidal input currents and regulate the DC output voltage. Current control is achieved using an inner current control loop that measures the phase current, I_n , and controls the inductor-neutral voltage, v_{n1} , to force the phase current to track its reference value. The current reference is provided by outer control loops that implement DC voltage and power factor control.

With space vector control, the inductor-neutral voltage is controlled as a vector quantity in the $\alpha\beta$ or dq domain. In this example, control is performed in the dq domain. The advantage of dq control is that AC quantities become DC quantities in the dq domain. Thus no tracking error exists when using a

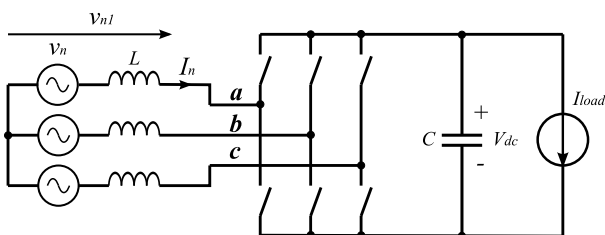


Fig. 1: Three-phase rectifier schematic diagram.

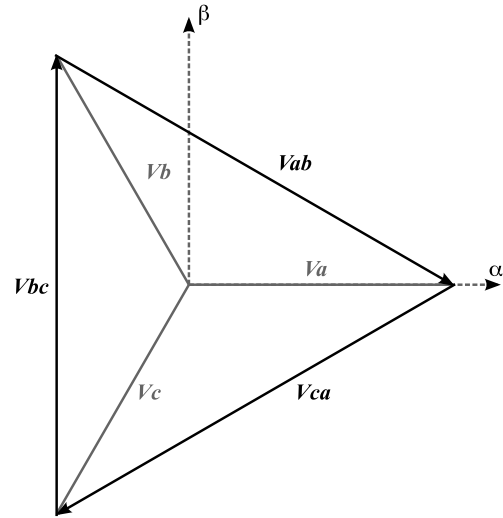


Fig. 2: Line voltage vectors in $\alpha\beta$ domain.

PI controller to regulate the AC input current.

2.1 Switching states

For the three-phase rectifier, the AC-side voltage required to induce sinusoidal currents through the inductors is first calculated. This reference vector, v_{n1} , is generated by time-averaging the available switching vectors. A switching vector is created on the AC-side of the rectifier by applying certain switch combinations to the rectifier bridge.

The line voltage vectors that are shown in Fig. 2 as vectors in the $\alpha\beta$ plane are added or subtracted to obtain the switching vector for a unique switching combination. Because the input lines cannot be shorted and continuous current must be maintained at the output, the switching states are restricted to eight combinations. Fig. 3 shows the two example switching states, (100) and (110), that are applied to the rectifier bridge. With switching state (100), $V_{ab} = V_{dc}$, $V_{bc} = 0$ and $V_{ca} = -V_{dc}$. The resultant switching vector for this state can be derived

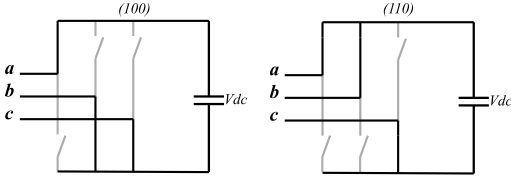


Fig. 3: Switching states (100) and (110).

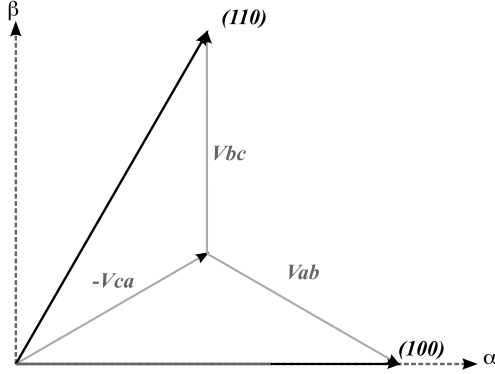


Fig. 4: Derivation of switching vectors (100) and (110) by summation of line voltage vectors.

by summing these line voltage vectors. The graphical derivation of switching vectors (100) and (110) is shown in Fig. 4. The complete set of switching vectors in the \$\alpha\beta\$ plane is shown in Fig. 5 along with the sectors encompassed by each pair of switching vectors. The vectors (000) and (111) are both zero voltage vectors.

2.2 Resolution of space vector \$\vec{v}_s\$

The space vector, \$\vec{v}_s\$, represents the inductor-neutral voltage, or AC-side voltage of the converter needed to achieve the desired current flow. This ref-

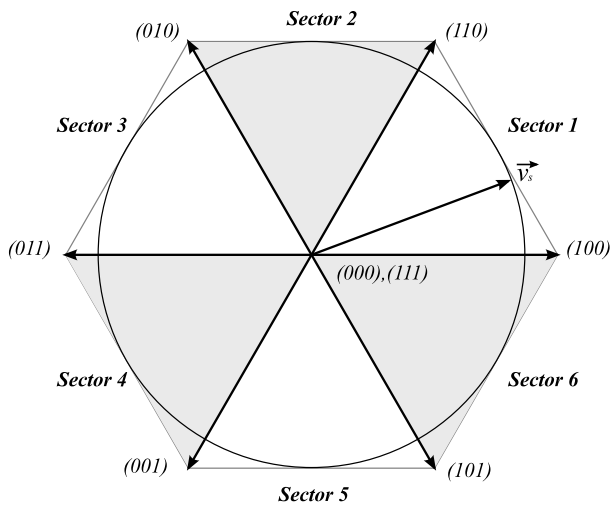


Fig. 5: Available switching vectors and sector definitions.

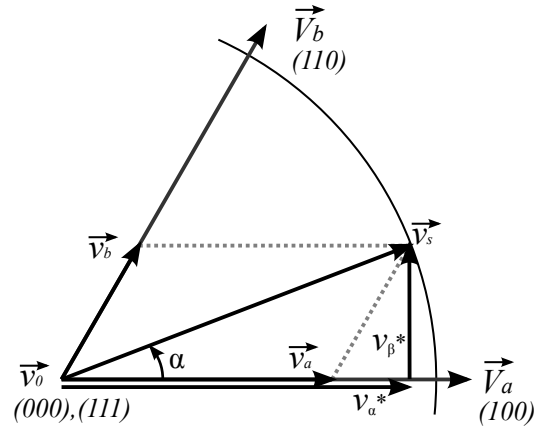


Fig. 6: Resolution of calculated vector \$\vec{v}_s\$ in sector 1 by time-averaging switching vectors \$\vec{V}_a\$, \$\vec{V}_b\$ and \$\vec{V}_0\$ over a switching cycle.

erence vector is generated by the current controller and is converted into \$\alpha\beta\$ coordinates. Thus the reference voltage vector is given by:

$$\vec{v}_s = v_{\alpha}^* + jv_{\beta}^* \quad (1)$$

The reference vector, \$\vec{v}_s\$, is resolved by time-averaging the available switching vectors, denoted as \$\vec{V}_a\$, \$\vec{V}_b\$ and \$\vec{V}_0\$. Fig. 6 shows the resolution of the vector \$\vec{v}_s\$ in sector 1.

$$\begin{aligned} \vec{v}_s &= \vec{v}_a + \vec{v}_b + \vec{v}_0 \\ &= \frac{t_a}{t_s} \vec{V}_a + \frac{t_b}{t_s} \vec{V}_b + \frac{t_0}{t_s} \vec{V}_0 \\ &= \tau_a \vec{V}_a + \tau_b \vec{V}_b + \tau_0 \vec{V}_0 \end{aligned} \quad (2)$$

Using the sine rule in conjunction with the relationship \$|\vec{V}_a| = |\vec{V}_b| = \frac{2}{3}V_{dc}\$, the relative on-times for the switching vectors \$\vec{V}_a\$, \$\vec{V}_b\$, and \$\vec{V}_0\$ can be calculated as follows:

$$\tau_a = \frac{\sqrt{3}|\vec{v}_s|}{V_{dc}} \sin\left(\frac{\pi}{3} - \alpha\right) \quad (3)$$

$$\tau_b = \frac{\sqrt{3}|\vec{v}_s|}{V_{dc}} \sin(\alpha) \quad (4)$$

$$\tau_0 = 1 - \tau_a - \tau_b \quad (5)$$

where \$\alpha\$ is the angle between \$\vec{v}_s\$ and the sector beginning.

2.3 Modulation strategies

The space vector \$\vec{v}_s\$ is generated by applying the switching vectors \$\vec{V}_a\$, \$\vec{V}_b\$ and \$\vec{V}_0\$ in a given se-

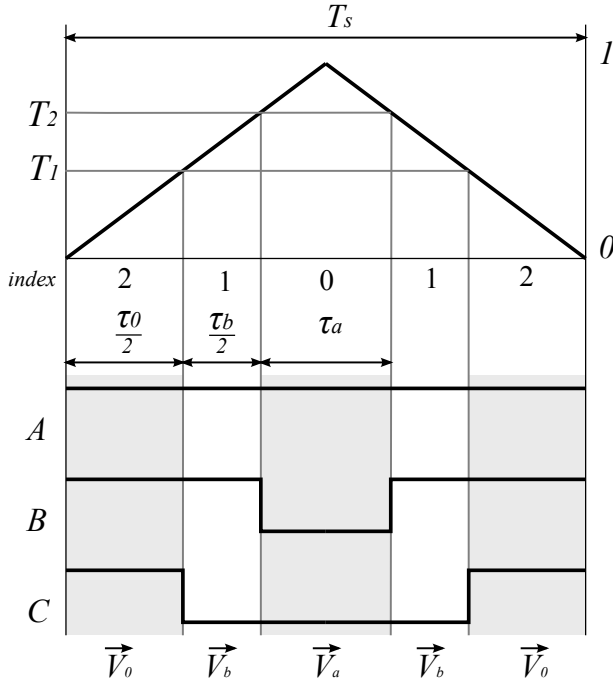


Fig. 7: Resolution of reference vector, \vec{v}_{s1} , in sector 1 using alternating zero vector modulation.

quence for their respective on-time values. The order in which the switching vectors are applied is referred to as the modulation strategy or switching sequence. Two popular modulation strategies have been implemented in the space vector modulator: an alternating zero vector strategy and a symmetrical modulation strategy.

2.3.1 Alternating zero vector modulation

With alternating zero vector modulation, only one of the two available zero vectors is used during a switching sequence, allowing one switch leg to be clamped to the positive or negative rail of the DC bus rather than switched. This modulation strategy therefore minimizes the number of switch transitions that occur during a single switching period. The generation of the reference vector in sector 1 using alternating zero vector modulation is graphically depicted in Fig. 7.

The space vector modulator is based on a state machine that sequences the available switching vectors. The internal operation of the space vector modulator is depicted in Fig. 8 and is implemented in a single PLECS C-Script block.

At the beginning of the switching cycle, the sector is detected based on the angle of the reference voltage, θ , and the relative on-times are calculated based on Eqs. (3)-(5). The PWM threshold values required for the state machine, T_1 and T_2 are calculated from the relative on-times as follows:

$$\begin{aligned} T_1 &= \tau_0 \\ T_2 &= \tau_0 + \tau_b \end{aligned} \quad (6)$$

During the switching cycle, the vector sequencer, based on a PWM generator, generates the switching vector transition times. The PWM generator is based on a highly-efficient implementation that does not require an iterative zero-crossing algorithm to detect the instants at which the PWM thresholds intersect the carrier wave. These transition times are calculated in advance at the beginning of the switching cycle and the state machine is only called by the solver at these instants.

To sequence the switching vectors, the two PWM output values are summed to provide an index signal, where $index \in [0, 1, 2]$. The index signal is used in conjunction with the sector value to extract the correct switching vector, $[V_0, V_a, V_b]$, that is stored in a lookup table. The switching vectors used for the alternating zero vector modulation strategy are listed in Table 1.

It can be seen from Fig. 7 that the switching sequence with alternating zero vector modulation comprises five time slices and results in four switch commutations. The zero vector is alternated in each switching sector to minimize the number of switching transitions. In odd-numbered sectors, the zero vector (111) is used and in even-numbered sectors, the zero vector is (000).

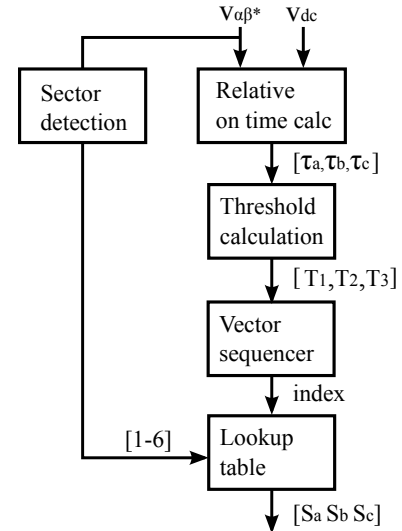


Fig. 8: Internal operation of space vector modulator. At the beginning of the cycle, the relative on-times and PWM thresholds are calculated. During the cycle, the PWM-based vector sequencer and lookup table are used to create the switching sequence.

Table 1: Switching vectors used for alternating zero vector modulation.

		index		
		0 = V_a	1 = V_b	2 = V_0
sector	1	100	110	111
	2	110	010	000
	3	010	011	111
	4	011	001	000
	5	001	101	111
	6	101	100	000

2.3.2 Symmetrical modulation

The generation of the reference vector in sector 1 for symmetrical modulation is graphically depicted in Fig. 9. The key difference between alternating zero vector and symmetrical modulation is that with symmetrical modulation, both zero vectors are applied during a single switching sequence. One zero vector is applied at the start and end of the switching cycle and the other zero vector is applied during the middle of the switching cycle. The resultant switching sequence comprises seven time slices.

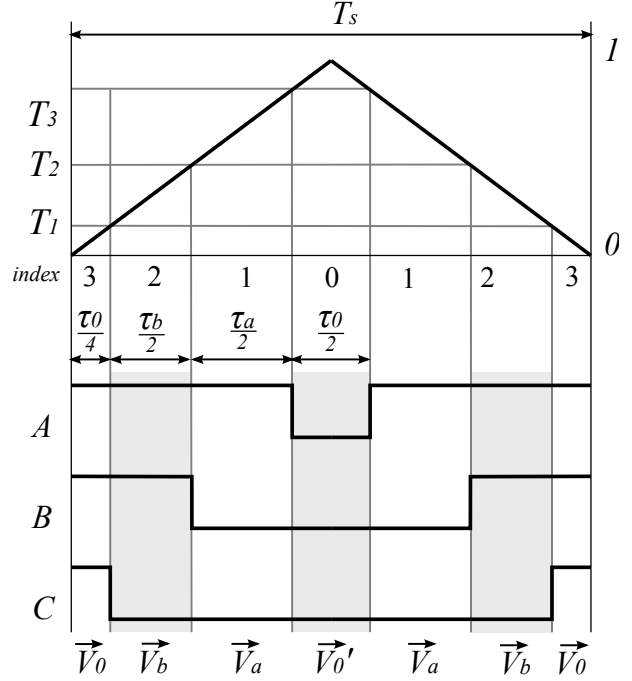
An additional PWM threshold is required for symmetrical modulation. The PWM thresholds are calculated as follows:

$$\begin{aligned} T_1 &= \frac{\tau_0}{2} \\ T_2 &= \frac{\tau_0}{2} + \tau_b \\ T_3 &= \frac{\tau_0}{2} + \tau_b + \tau_a \end{aligned} \quad (7)$$

The switching losses with the symmetrical modulation are higher than the losses with alternating zero vector modulation because the number of switch commutations is higher. However this modulation strategy has one key advantage. Due to the use of both zero vectors, the generated output voltage has a lower THD content.

3 Control Strategy

The block diagram of the rectifier controller is shown in Fig. 10. The inner control loop comprises a P+I dq current controller and an outer DC voltage control loop regulates the DC output of the rectifier by adjusting the reference current, i_d^* . Power factor control could be implemented through control of i_q^* , but in this example, i_q^* is set to zero to achieve unity power factor operation.


 Fig. 9: Resolution of reference vector, \vec{v}_s , in sector 1 using symmetrical modulation.

3.1 Current control

The current control loop, implemented in the dq domain, calculates the inductor-neutral voltage voltage that is needed to generate the required input current. The dq transformation that is performed on the line currents is given in the Appendix. The d axis is aligned with the supply voltage angle, θ . The dq control loop has a structure that uses PI controllers to determine the voltage drop needed across the input inductor at the required operating point. The inductor-neutral voltage, $v_{d1}^* + jv_{q1}^*$, is then calculated by subtracting the inductor voltage drop from the input voltage. The dq current control structure is derived as follows:

Neglecting the line resistance, the voltage at the rectifier AC terminals is given by:

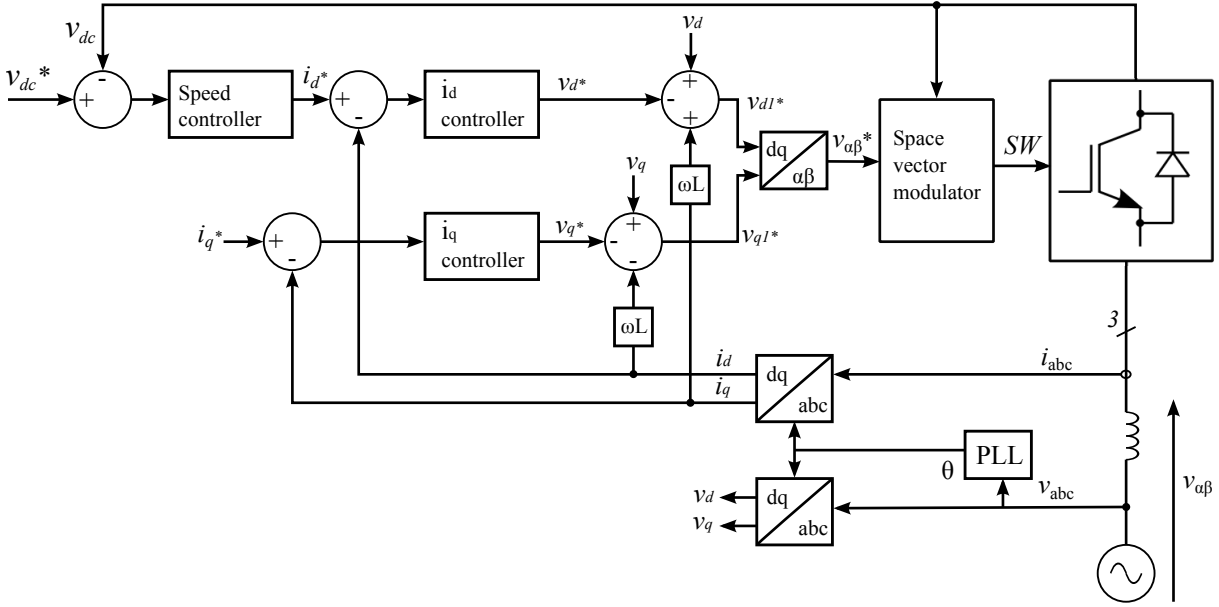
$$v_{n1} = v_n - L \frac{di_n}{dt} \quad (8)$$

where the subscript n denotes phase A, B or C. Transforming this equation to the dq domain yields

$$v_{d1} = v_d - L \frac{di_d}{dt} + \omega L i_q \quad (9)$$

$$v_{q1} = v_q - L \frac{di_q}{dt} - \omega L i_d \quad (10)$$

where the $\omega L i_{d,q}$ terms are cross-coupling terms introduced by the dq transformation. The reference


 Fig. 10: Block diagram of rectifier control system with dq current control.

values of the rectifier AC-side voltages can therefore be described by the following equations:

$$v_{d1}^* = v_d + \omega L i_q - v_{Ld} \quad (11)$$

$$v_{q1}^* = v_q - \omega L i_d - v_{Lq} \quad (12)$$

where the $v_{Ld,q}$ terms are provided by the PI current controllers.

Thus the task of the dq current controllers is to control the appropriate inductor voltages that force dq currents to track their reference values. It should be noted that under steady state supply voltage conditions, $v_q = 0$ and therefore may be neglected from Eq. (12).

The total output of the current control loop, $v_{d1}^* + jv_{q1}^*$ represents the voltage across the AC supply and AC inductor, which is equivalent to the AC terminal voltage of the converter. This reference value is converted into an $\alpha\beta$ value as required by the space vector modulator.

3.2 Voltage control

The real and reactive power in the dq domain is given by:

$$P = v_d i_d + v_q i_q \quad (13)$$

$$Q = v_d i_q - v_q i_d \quad (14)$$

Since the d axis is aligned with the supply voltage vector as shown in Fig. 6, v_d is constant and v_q is

zero. Thus the real and reactive power can be controlled by controlling i_d and i_q , respectively.

For the example rectifier, the power factor is set to unity by setting $Q = 0$; therefore, $i_q^* = 0$. The voltage level of the DC bus is regulated by controlling the real power flow with i_d^* .

Appendices

A Transformations

$$\begin{bmatrix} y_\alpha \\ y_\beta \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{bmatrix} \cdot \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}$$

$$\begin{bmatrix} y_\alpha \\ y_\beta \end{bmatrix} = \begin{bmatrix} \cos \omega_1 t & -\sin \omega_1 t \\ \sin \omega_1 t & \cos \omega_1 t \end{bmatrix} \cdot \begin{bmatrix} x_d \\ x_q \end{bmatrix}$$

$$\begin{bmatrix} y_d \\ y_q \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos \varphi & -\sin \varphi \\ \cos(\varphi - 120^\circ) & -\sin(\varphi - 120^\circ) \\ \cos(\varphi + 120^\circ) & -\sin(\varphi + 120^\circ) \end{bmatrix}^T \cdot \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}$$